Modeling, Control, and Simulation of a Thermal Process Implemented with Two-Thermal Tanks Connected in Series

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***Abstract*—This paper describes the modeling and simulation of a thermal process, built by two thermal tanks connected in series. The process involves processing a fluid by mixing it with other streams, with different thermal properties, in order to reach a desired outlet temperature, by means of a control system.**

The proposed control system is based on two regulation goals. On the one hand, it is necessary to control the outlet temperature of the fluid to be processed and on the other hand, it is required to regulate the levels of both tanks. It is identified then, that the control system is multivariable, since there are more than two references and actuation signals.

A crucial stage of the paper is the mathematical modeling of the system, obtaining a set of non-linear differential equations, which are linearized by applying the perturbation and Taylor series expansion technique.

Finally, the process is dynamically simulated, verifying its proper operation.

Keywords*—*Mass- and energy-balance, multivariable controller, non-linear equations, thermal process, simulation.

I. INTRODUCTION

Many people in my professional environment, in the area of process control, ask me, "hey, what good is all this paraphernalia of process modeling and control, if when I get hired in a company as a control engineer I will not use it and I will not remember it either? And in a way, they are somewhat fitting. Let's see an example of why this question is valid using as a basis of foundation, the typical tasks of a control engineer in a continuous processing plant of some raw material, for example, cellulose. When a control engineer is hired in a plant of this type, its main mission is to keep the control system as operational - in optimal operation - as possible, to minimize failures based on hardware or software problems. In other words, its mission is to properly maintain the factory control system, i.e. design and implement maintenance plans to minimize the chances (statistics) of failures. At the same time, it has a responsibility of vital importance; it becomes a "private investigator process, hired to find traces and clues, using the tools provided by the control system and thereby elucidate the reason for the failures produced in any process in the factory. However, it may also have a secondary responsibility more closely linked to the area of process control engineering, which is based on the design of algorithms and/or controls diagrams, as well as continuous improvement in the tuning of PID compensators so massively used in plants of this type, even today. The control engineer does not require the use of process modeling techniques or analytical tools for designing and tuning PID compensators so that it can successfully fulfill the tasks entrusted above.

Now, if we focus on the role of an investigator of the factory process, which the engineer must fulfill, and according to my experience, being a good investigator is almost a primary requirement for a control engineer, since there will always be failures in the processes, due to different factors (operational and maintenance) and the less time the engineer takes to find the failures, the faster the process will be started up. It is also known that being a good researcher depends on several factors, but of which I will focus on two: 1) the educational background that has the engineer, on issues of process engineering and control and 2) how persevering the engineer to develop as a good researcher, which includes many hours in training, often empirically - witnessing failures and studying the techniques used by more experienced control engineers when analyzing them - following the known learning curve.

If we analyze in some detail the two factors mentioned above, we can find a common point between them and that has a predominance with the time it takes the engineer, to train as a good researcher, this is the knowledge that the engineer of the dynamics of the processes involved in the plant. For both cases, if the engineer knows the dynamics of the processes in question, he will be able to analyze in a better way the variables involved in the processes, taking less time to find the fault. And of course, knowledge of process dynamics is directly related to the training you have in process modeling and design of control systems.

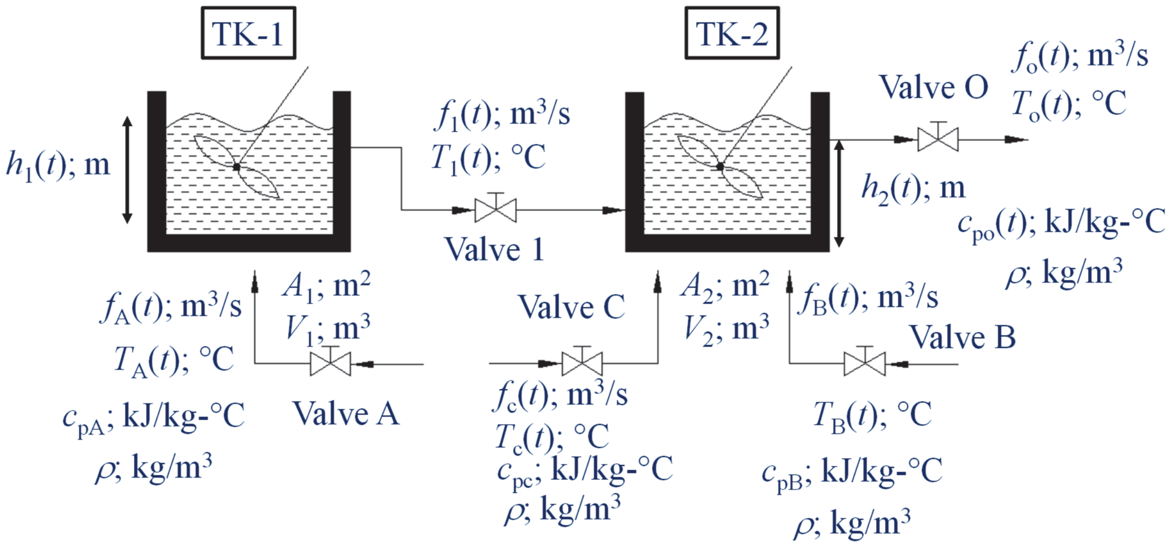
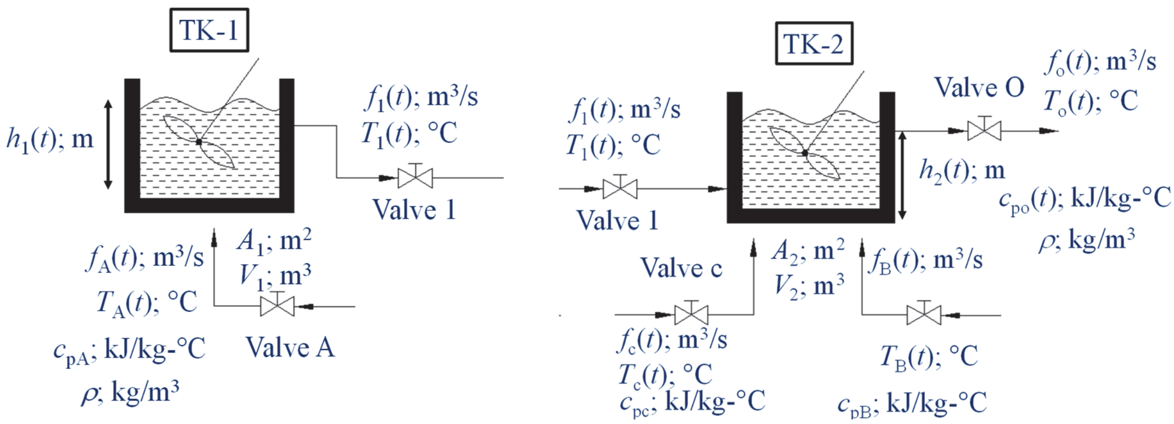


Fig. 1. Thermal process implemented by two tanks connected in series.



(a) (b)

Fig. 2. Process control volumes. (a) TK-1 control volume. (b) TK-2 control volume.

The following white paper aims to present the stages involved in the modeling of an industrial process and then simulate its behavior using simulation software.

II. Process Description

The process to be studied consists of a thermal process composed of two tanks connected in series and it was presented by [1]. The process is shown in Fig. 1. The first tank (TK-1) serves as a buffer and the second tank (TK-1) generates the mixing of the fluid coming from TK-1 with two feed streams of different heat capacities. It should be noted that each of the thermal streams involved in the process have different heat capacities, but that all their densities are approximately equal.

The TK-1 has a feed pipe which is connected to the TK-1 through a control valve labeled valve A. Through it flows a fluid associated with a flow rate $f_A(t)$ at a temperature $T_A(t)$ and with a heat capacity c_{pA}. Also, TK-1 has a discharge pipe, which allows its connection with TK-2 through the manual valve labeled valve 1. Through this pipe flows a fluid $f_1(t)$ at a temperature $T_1(t)$. Finally, the level of TK-1 is labeled $h_1(t)$.

TK-2 has three feed pipes. One of them is the discharge of TK-1 related to the flow $f_1(t)$, then, there is a second feed stream which is connected to TK-2 through the control valve C and relates to a flow $f_C(t)$ and which is at a temperature $T_C(t)$. In addition, this current is associated with a heat capacity c_{pC}. Finally, the third feed stream supplied to the TK-2, through the control valve valve B, has a flow $f_B(t)$ at a temperature $T_B(t)$ and with a heat capacity c_{pB}. Then the discharge of the TK-2 corresponds to the flow $f_o(t)$ at an outlet temperature $T_o(t)$ and with a heat capacity of c_{po}. The outlet pipe is connected to the outside of the process through a

manual valve (valve O). The TK-2 level is designated as $h_2(t)$.

It is assumed that the process is adiabatic, that there are no dead times associated with the pipes. In addition, the process is assumed to be atmospheric. All flow rates are volumetric measured in m^3/s, the heat capacities associated with the inlet streams to the tanks are measured at constant pressure and their units are kJ/kg-°C. Finally, densities are measured in kg/m^3. The tanks (TK-1 and TK-2) have the volumes V_1 and V_2 respectively, measured in m^3.

III. Process Modeling

The modeling of the process was developed considering the mass and energy balances applied to each of the tanks [1]–[3]. It should be noted that each of the tanks act as the control volumes to be studied. Thus, when developing the mass balance around TK-1 and TK-2 (see Fig. 2), the dynamic equations (1) and (2) are obtained for each tank respectively.

$$f_{\mathrm{A}}(t) - f_1(t) = A_1 \cdot \frac{\mathrm{d}h_1(t)}{\mathrm{d}t} \tag{1}$$

$$f_{\mathrm{B}}(t) + f_{\mathrm{c}}(t) - f_1(t) = A_2 \cdot \frac{\mathrm{d}h_2(t)}{\mathrm{d}t}, \tag{2}$$

where A_1 and A_2 are the cross sections of TK-1 and TK-2 respectively, measured in m^2.

Then by developing the energy balances around TK-1 and TK-2, the dynamic energy equations of each tank are obtained and presented in (3) and (4) respectively [1].

$$\begin{gathered} f_{\mathrm{A}}(t) \cdot T_{\mathrm{A}}(t) - \\ -f_1(t) \cdot T_1(t) = V_1 \cdot \frac{\mathrm{d}T_1(t)}{\mathrm{d}t} \end{gathered} \tag{3}$$

$$\begin{gathered} c_{\mathrm{p_A}} \cdot f_1(t) \cdot T_1(t) + \\ +c_{\mathrm{p_B}} \cdot f_{\mathrm{B}}(t) \cdot T_{\mathrm{B}}(t) + \\ +c_{\mathrm{p_C}} \cdot f_{\mathrm{C}}(t) \cdot T_{\mathrm{C}}(t) - \\ -c_{\mathrm{p_o}}(t) \cdot f_{\mathrm{o}}(t) \cdot T_{\mathrm{o}}(t) = \\ = c_{\mathrm{p_o}}(t) \cdot V_2 \cdot \frac{\mathrm{d}T_{\mathrm{o}}(t)}{\mathrm{d}t}. \end{gathered} \tag{4}$$

It should be noted that since streams with different heat capacities are found in TK-2, it is necessary to use the mixing rules [3]. To apply this rule, it is assumed that the volumes and cross sections of the two tanks are equal, i.e., $V_1 = V_2$ and $A_1 = A_2$. Therefore, the model of $c_{\mathrm{po}}(t)$ is presented in (5).

$$\begin{gathered} c_{\mathrm{p_o}}(t) = \frac{h_1(t)}{h_1(t)+h_2(t)} \cdot c_{\mathrm{p_A}} + \\ + \frac{h_2(t)}{h_1(t)+h_2(t)} \cdot \left(c_{\mathrm{p_B}} + c_{\mathrm{p_C}}\right). \end{gathered} \tag{5}$$

The coupling elements between equations (1)–(4) are for the manual valves. Considering the atmospheric characteristic of the process, $f_1(t)$ is expressed as [1]

$$f_1(t) = c'_{\mathrm{v_1}} \cdot \sqrt{h_1(t) - h_2(t)} \tag{6}$$

$$c'_{\mathrm{v_1}} = c_{\mathrm{v_1}} \cdot \sqrt{\frac{\rho \cdot g}{G_{\mathrm{f}}}}. \tag{7}$$

Here g and G_{f} are the gravitational acceleration and the specific density of the fluid. c_{v1} is the characteristic of valve 1 (see Fig. 2(a)). On the other hand, $f_{\mathrm{o}}(t)$ is modeled as [1]:

$$f_{\mathrm{o}}(t) = c'_{\mathrm{v_O}} \cdot \sqrt{h_2(t)} \tag{8}$$

$$c'_{\mathrm{v_O}} = c_{\mathrm{v_O}} \cdot \sqrt{\frac{\rho \cdot g}{G_{\mathrm{f}}}}, \tag{9}$$

c_{vo} is the characteristic of valve O. Finally by replacing equations (5)–(9) in (1)–(4), the process model is obtained and presented in (10). The main characteristic of the process model is its nonlinear nature, due to the influence of the manual valves (valve 1 and valve O). As mentioned in previous paragraphs, these valves play a coupling role between equations (1)–(4).

To design a suitable control system for the process, it is convenient to use conventional control techniques, which mostly deal with linear output feedback compensators, to name a few, PIs or PIDs or feedback state controllers [4], [5]. A necessary condition for designing these types of compensators and controllers is that the plant is linear. Therefore, it is essential to linearize the model in (10).

The linearization of the model involves the use of the Taylor series expansion in each of the equations in (10), incorporating the deviation variables [4], [5]. To do so, it is required, first, to find the equilibrium points of the model, zeroing the derivatives and replacing the variables by their capital letters with the superscript ss. The model in (11) corresponds to the model of the steady state process. On the other hand, the transposed vector that gathers the equilibrium points of the model is defined as $\mathbf{q}^{\mathbf{ss}} = [F_{\mathrm{A}}^{\mathrm{ss}}, \mathrm{F}_{\mathrm{B}}^{\mathrm{ss}}, F_{\mathrm{C}}^{\mathrm{ss}}, H_1^{\mathrm{ss}}, H_2^{\mathrm{ss}}, T_1^{\mathrm{ss}}, T_{\mathrm{O}}^{\mathrm{ss}}, T_{\mathrm{A}}^{\mathrm{ss}}, T_{\mathrm{B}}^{\mathrm{ss}}, T_{\mathrm{C}}^{\mathrm{ss}}]^{\mathrm{T}}$.

Finally, the system of differential equations given in (12) is the linear model of the process. The variables with hats are the deviation variables. Each of the constants involved in the equations in (12) are defined in (13). Also the Taylor series expansions associated with (12) are described in (14), where hot means high-order terms. Naturally, it is assumed that

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$$\begin{cases} A_1 \cdot \dfrac{\mathrm{d}h_1(t)}{\mathrm{d}t} = f_\mathrm{A}(t) - c'_{\mathrm{v}_1} \cdot \sqrt{h_1(t) - h_2(t)} \\ A_2 \cdot \dfrac{\mathrm{d}h_2(t)}{\mathrm{d}t} = f_\mathrm{B}(t) + f_\mathrm{C}(t) + c'_{\mathrm{v}_1} \cdot \sqrt{h_1(t) - h_2(t)} - c'_{\mathrm{v}_\mathrm{O}} \cdot \sqrt{h_2(t)} \\ V_1 \cdot \dfrac{\mathrm{d}T_1(t)}{\mathrm{d}t} = f_\mathrm{A}(t) \cdot T_\mathrm{A}(t) - c'_{\mathrm{v}_1} \cdot T_1(t) \cdot \sqrt{h_1(t) - h_2(t)} \\ V_2 \cdot \dfrac{\mathrm{d}T_\mathrm{o}(t)}{\mathrm{d}t} = \dfrac{c_{\mathrm{p}_\mathrm{B}} \cdot f_\mathrm{B}(t) \cdot T_\mathrm{B}(t)}{c_{\mathrm{p}_\mathrm{o}}(t)} + \dfrac{c_{\mathrm{p}_\mathrm{C}} \cdot f_\mathrm{C}(t) \cdot T_\mathrm{C}(t)}{c_{\mathrm{p}_\mathrm{o}}(t)} + \\ + \dfrac{c_{\mathrm{v}_1} \cdot c_{\mathrm{p}_\mathrm{A}} \cdot T_1(t) \cdot \sqrt{h_1(t) - h_2(t)}}{c_{\mathrm{p}_\mathrm{o}}(t)} \end{cases} \tag{10}$$

$$\begin{cases} F_\mathrm{A}^\mathrm{ss} - c'_{\mathrm{v}_1} \cdot \sqrt{H_1^\mathrm{ss} - H_2^\mathrm{ss}} = 0 \\ F_\mathrm{B}^\mathrm{ss} + F_\mathrm{c}^\mathrm{ss} + c'_{\mathrm{v}_1} \cdot \sqrt{H_1^\mathrm{ss} - H_2^\mathrm{ss}} - c'_{\mathrm{v}_2} \cdot \sqrt{H_2^\mathrm{ss}} = 0 \\ F_\mathrm{A}^\mathrm{ss} \cdot T_\mathrm{A}^\mathrm{ss} - c'_{\mathrm{v}_1} \cdot T_1^\mathrm{ss} \cdot \sqrt{H_1^\mathrm{ss} - H_2^\mathrm{ss}} = 0 \\ \dfrac{c_{\mathrm{p}_\mathrm{B}} \cdot F_\mathrm{B}^\mathrm{ss} \cdot T_\mathrm{B}^\mathrm{ss}}{{c_{\mathrm{p}_\mathrm{o}}}^\mathrm{ss}} + \dfrac{c_{\mathrm{p}_\mathrm{c}} \cdot F_\mathrm{c}^\mathrm{ss} \cdot T_\mathrm{c}^\mathrm{ss}}{{c_{\mathrm{p}_\mathrm{o}}}^\mathrm{ss}} + \dfrac{c_{\mathrm{v}_1} \cdot c_{\mathrm{p}_\mathrm{A}} \cdot T_1^\mathrm{ss} \cdot \sqrt{H_1^\mathrm{ss} - H_2^\mathrm{ss}}}{{c_{\mathrm{p}_\mathrm{o}}}^\mathrm{ss}} = 0 \end{cases} \tag{11}$$

$$\begin{cases} A_1 \cdot \dfrac{\mathrm{dh}_1(t)}{\mathrm{d}t} \approx K_{11} \cdot \hat{f}_\mathrm{A}(t) + K_{21} \cdot \hat{\mathrm{h}}_1(t) + K_{31} \cdot \hat{\mathrm{h}}_2(t) + K_{41} \cdot \hat{\mathrm{h}}_2(t) \\ A_2 \cdot \dfrac{\mathrm{dh}_2(t)}{\mathrm{d}t} \approx K_{12} \cdot \hat{f}_\mathrm{B}(t) + K_{22} \cdot \hat{\mathrm{h}}_1(t) + K_{32} \cdot \hat{\mathrm{h}}_2(t) \\ V_1 \cdot \dfrac{\mathrm{d}T_1(t)}{\mathrm{d}t} \approx K_{13} \cdot \hat{f}_\mathrm{A}(t) + K_{23} \cdot \hat{\mathrm{h}}_1(t) + K_{33} \cdot \hat{\mathrm{h}}_2(t) + K_{43} \cdot \hat{T}_\mathrm{A}(t) + K_{53} \cdot \hat{T}_1(t) \\ V_2 \cdot \dfrac{\mathrm{d}T_\mathrm{o}(t)}{\mathrm{d}t} \approx K_{14} \cdot \hat{f}_\mathrm{B}(t) + K_{24} \cdot \hat{f}_\mathrm{c}(t) + K_{34} \cdot \hat{\mathrm{h}}_1(t) + K_{44} \cdot \hat{\mathrm{h}}_2(t) + K_{54} \cdot \hat{T}_\mathrm{B}(t) + \\ + K_{64} \cdot \hat{T}_\mathrm{c}(t) + K_{74} \cdot \hat{T}_1(t). \end{cases} \tag{12}$$

$$\mathbf{z} = [z_1, z_2, \ldots, z_4]^\mathrm{T}, \tag{18}$$

$$\mathbf{A} = \left[\frac{\partial \mathrm{z}}{\partial \mathrm{x}}\right]_{\mathbf{q_{ss}}}$$

$$\mathbf{B} = \left[\frac{\partial \mathrm{z}}{\partial \mathrm{u}}\right]_{\mathbf{q_{ss}}}$$

$$\mathbf{C} = \left[\frac{\partial \mathrm{y}}{\partial \mathrm{x}}\right]_{\mathbf{q_{ss}}}$$

$$\mathbf{D} = \left[\frac{\partial \mathrm{y}}{\partial \mathrm{u}}\right]_{\mathbf{q_{ss}}}, \tag{19}$$

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A} \cdot \mathbf{x} + \mathbf{B} \cdot \mathbf{u} \\ \hat{\mathbf{y}} = \mathbf{C} \cdot \mathbf{x} + \mathbf{D} \cdot \mathbf{u} \end{cases} \tag{20}$$

the deviation variables are much smaller than the absolute variables. The definitions of the deviation variables are given in (15).

A convenient way to express the linear model of the process is through the state-space model [4], [5] According to (12), the state variables, as input and output variables, are defined in the form of vectors and shown in (16). In this case, the states are assumed to be equal to the inputs. The definition of the vectors are $\{\mathbf{x},\mathbf{y}\} \in \{\mathbb{R}^4\}$ and $\mathbf{u} \in \{\mathbb{R}^6\}$. Now, grouping the linear dynamic equations present in (12), in the form of a vector of functions, as shown in (17). The system in (17) can be more compactly described as in (18) where $\mathbf{z} \in \{\mathbb{R}^4\}$. From the definitions in (16) and (18) the matrices of the state-space model are found and defined in (19).

The matrices in (19), i.e., **A**, **B**, **C**, and **D** are the state, input, output, and direct transmission matrices respectively [4]. Now $\{\mathbf{A},\mathbf{C}\} \in \mathbb{M}_{4\mathrm{x}4}\{\mathbb{R}\}$ and $\{\mathbf{B}, \mathbf{D}\} \in \mathbb{M}_{4\mathrm{x}6}\{\mathbb{R}\}$. Finally, the state-space model is described in (20).

$$K_{11} = \left.\frac{\partial \mathrm{g}_1}{\partial f_{\mathrm{A}}(t)}\right|_{\mathbf{q_{ss}}}, K_{12} = \left.\frac{\partial \mathrm{g}_1}{\partial h_1(t)}\right|_{\mathbf{q_{ss}}}, K_{13} = \left.\frac{\partial \mathrm{g}_1}{\partial h_2(t)}\right|_{\mathbf{q_{ss}}}$$

$$K_{12} = \left.\frac{\partial \mathrm{g}_2}{\partial f_{\mathrm{B}}(t)}\right|_{\mathbf{q_{ss}}}, K_{22} = \left.\frac{\partial \mathrm{g}_2}{\partial h_1(t)}\right|_{\mathbf{q_{ss}}}, K_{32} = \left.\frac{\partial \mathrm{g}_2}{\partial h_2(t)}\right|_{\mathbf{q_{ss}}}$$

$$K_{13} = \left.\frac{\partial \mathrm{g}_3}{\partial f_{\mathrm{A}}(t)}\right|_{\mathbf{q_{ss}}}, K_{23} = \left.\frac{\partial \mathrm{g}_3}{\partial h_1(t)}\right|_{\mathbf{q_{ss}}}, K_{33} = \left.\frac{\partial \mathrm{g}_3}{\partial h_2(t)}\right|_{\mathbf{q_{ss}}}, K_{43} = \left.\frac{\partial \mathrm{g}_3}{\partial T_{\mathrm{A}}(t)}\right|_{\mathbf{q_{ss}}}, K_{53} = \left.\frac{\partial \mathrm{g}_3}{\partial T_1(t)}\right|_{\mathbf{q_{ss}}}$$

$$K_{14} = \left.\frac{\partial \mathrm{g}_4}{\partial f_{\mathrm{B}}(t)}\right|_{\mathbf{q_{ss}}}, K_{24} = \left.\frac{\partial \mathrm{g}_4}{\partial f_{\mathrm{C}}(t)}\right|_{\mathbf{q_{ss}}}, K_{34} = \left.\frac{\partial \mathrm{g}_4}{\partial h_1(t)}\right|_{\mathbf{q_{ss}}}, K_{44} = \left.\frac{\partial \mathrm{g}_4}{\partial h_2(t)}\right|_{\mathbf{q_{ss}}}, K_{54} = \left.\frac{\partial \mathrm{g}_4}{\partial T_{\mathrm{B}}(t)}\right|_{\mathbf{q_{ss}}},$$

$$K_{64} = \left.\frac{\partial \mathrm{g}_4}{\partial T_{\mathrm{C}}(t)}\right|_{\mathbf{q_{ss}}}, K_{74} = \left.\frac{\partial \mathrm{g}_4}{\partial T_1(t)}\right|_{\mathbf{q_{ss}}}. \tag{13}$$

$$\begin{cases} A_1 \cdot \frac{\mathrm{d}h_1(t)}{\mathrm{d}t} = \left.\frac{\partial \mathrm{g}_1}{\partial f_{\mathrm{A}}(t)}\right|_{\mathbf{q_{ss}}} \cdot (f_{\mathrm{A}}(t) - F_{\mathrm{A}}^{\mathrm{ss}}) + \left.\frac{\partial \mathrm{g}_1}{\partial h_1(t)}\right|_{\mathbf{q_{ss}}} \cdot (h_1(t) - H_1^{\mathrm{ss}}) + \\ \qquad + \left.\frac{\partial \mathrm{g}_1}{\partial h_2(t)}\right|_{\mathbf{q_{ss}}} \cdot (h_2(t) - H_2^{\mathrm{ss}}) + \mathrm{hot} \\ A_2 \cdot \frac{\mathrm{d}h_2(t)}{\mathrm{d}t} = \left.\frac{\partial \mathrm{g}_2}{\partial f_{\mathrm{B}}(t)}\right|_{\mathbf{q_{ss}}} \cdot (f_{\mathrm{B}}(t) - F_{\mathrm{B}}^{\mathrm{ss}}) + \left.\frac{\partial \mathrm{g}_2}{\partial h_1(t)}\right|_{\mathbf{q_{ss}}} \cdot (h_1(t) - H_1^{\mathrm{ss}}) + \\ \qquad + \left.\frac{\partial \mathrm{g}_2}{\partial h_2(t)}\right|_{\mathbf{q_{ss}}} \cdot (h_2(t) - H_2^{\mathrm{ss}}) + \mathrm{hot} \\ V_1 \cdot \frac{\mathrm{d}T_1(t)}{\mathrm{d}t} = \left.\frac{\partial \mathrm{g}_3}{\partial f_{\mathrm{A}}(t)}\right|_{\mathbf{q_{ss}}} \cdot (f_{\mathrm{A}}(t) - F_{\mathrm{A}}^{\mathrm{ss}}) + \left.\frac{\partial \mathrm{g}_3}{\partial h_1(t)}\right|_{\mathbf{q_{ss}}} \cdot (h_1(t) - H_1^{\mathrm{ss}}) + \\ \qquad + \left.\frac{\partial \mathrm{g}_3}{\partial h_2(t)}\right|_{\mathbf{q_{ss}}} \cdot (h_2(t) - H_2^{\mathrm{ss}}) + \left.\frac{\partial \mathrm{g}_3}{\partial T_{\mathrm{A}}(t)}\right|_{\mathbf{q_{ss}}} \cdot (T_{\mathrm{A}}(t) - T_{\mathrm{A}}^{\mathrm{ss}}) + \\ \qquad + \left.\frac{\partial \mathrm{g}_3}{\partial T_1(t)}\right|_{\mathbf{q_{ss}}} \cdot (T_1(t) - T_1^{\mathrm{ss}}) + \mathrm{hot} \\ V_2 \cdot \frac{\mathrm{d}T_{\mathrm{o}}(t)}{\mathrm{d}t} = \left.\frac{\partial \mathrm{g}_4}{\partial f_{\mathrm{B}}(t)}\right|_{\mathbf{q_{ss}}} \cdot (f_{\mathrm{B}}(t) - F_{\mathrm{B}}^{\mathrm{ss}}) + \left.\frac{\partial \mathrm{g}_4}{\partial f_{\mathrm{c}}(t)}\right|_{\mathbf{q_{ss}}} \cdot (f_{\mathrm{c}}(t) - F_{\mathrm{c}}^{\mathrm{ss}}) + \\ \qquad \left.\frac{\partial \mathrm{g}_4}{\partial h_1(t)}\right|_{\mathbf{q_{ss}}} \cdot (h_1(t) - H_1^{\mathrm{ss}}) + \left.\frac{\partial \mathrm{g}_4}{\partial h_2(t)}\right|_{\mathbf{q_{ss}}} \cdot (h_2(t) - H_2^{\mathrm{ss}}) + \\ \qquad + \left.\frac{\partial \mathrm{g}_4}{\partial T_{\mathrm{B}}(t)}\right|_{\mathbf{q_{ss}}} \cdot (T_{\mathrm{B}}(t) - T_{\mathrm{B}}^{\mathrm{ss}}) + \left.\frac{\partial \mathrm{g}_4}{\partial T_{\mathrm{c}}(t)}\right|_{\mathbf{q_{ss}}} \cdot (T_{\mathrm{c}}(t) - T_{\mathrm{c}}^{\mathrm{ss}}) + \\ \qquad + \left.\frac{\partial \mathrm{g}_4}{\partial T_1(t)}\right|_{\mathbf{q_{ss}}} \cdot (T_1(t) - T_1^{\mathrm{ss}}) + \mathrm{hot.} \end{cases} \tag{14}$$

$$\begin{cases} \hat{f}_{\mathrm{A}}(t) = f_{\mathrm{A}}(t) - F_{\mathrm{A}}^{\mathrm{ss}} \\ \hat{f}_{\mathrm{B}}(t) = f_{\mathrm{B}}(t) - F_{\mathrm{B}}^{\mathrm{ss}} \\ \hat{h}_1(t) = h_1(t) - H_1^{\mathrm{ss}} \\ \hat{h}_2(t) = h_2(t) - H_2^{\mathrm{ss}} \\ \hat{T}_A(t) = T_{\mathrm{A}}(t) - T_{\mathrm{A}}^{ss} \\ \hat{T}_B(t) = T_{\mathrm{B}}(t) - T_{\mathrm{B}}^{ss} \\ \hat{T}_c(t) = T_{\mathrm{C}}(t) - T_{\mathrm{C}}^{ss} \\ \hat{T}_1(t) = T_1(t) - T_1^{ss}. \end{cases} \tag{15}$$

$$\begin{cases} \mathbf{x} = \mathbf{y} = \left[\hat{h}_1(t), \hat{h}_2(t), \hat{T}_1(t), \hat{T}_{\mathrm{o}}(t)\right]^{\mathrm{T}} \\ \mathbf{u} = \left[\hat{f}_{\mathrm{A}}(t), \hat{f}_{\mathrm{B}}(t), \hat{f}_{\mathrm{c}}(t), \hat{T}_{\mathrm{A}}(t), \hat{T}_{\mathrm{B}}(t), \hat{T}_{\mathrm{c}}(t)\right]^{\mathrm{T}}. \end{cases} \tag{16}$$

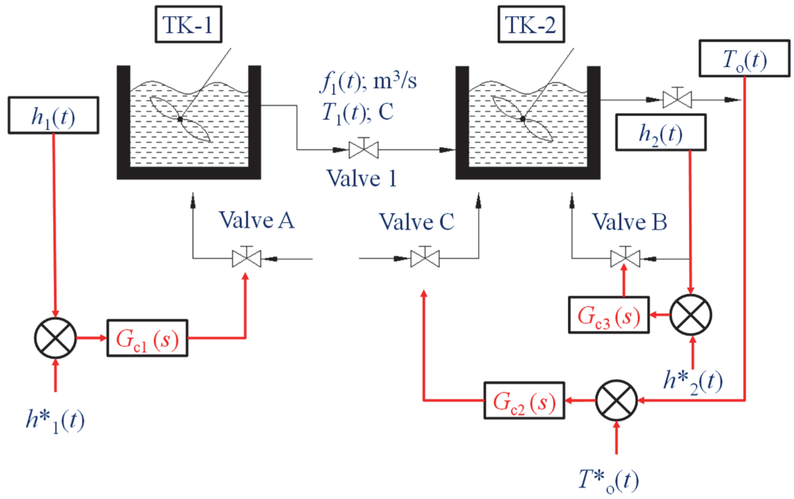

Fig. 3. PI&D of the thermal process. The compensators are identified, as are the measured process variables and the actuation variables.

$$\begin{cases} z_1 = K_{11} \cdot \hat{f}_A(t) + K_{21} \cdot \hat{h}_1(t) + K_{31} \cdot \hat{h}_2(t) + K_{41} \cdot \hat{h}_2(t) \\ z_2 = K_{12} \cdot \hat{f}_B(t) + K_{22} \cdot \hat{h}_1(t) + K_{32} \cdot \hat{h}_2(t) \\ z_3 = K_{13} \cdot \hat{f}_A(t) + K_{23} \cdot \hat{h}_1(t) + K_{33} \cdot \hat{h}_2(t) + K_{43} \cdot \hat{T}_A(t) + K_{53} \cdot \hat{T}_1(t) \\ z_4 = K_{14} \cdot \hat{f}_B(t) + K_{24} \cdot \hat{f}_c(t) + K_{34} \cdot \hat{h}_1(t) + K_{44} \cdot \hat{h}_2(t) + K_{54} \cdot \hat{T}_B(t) + \\ \quad + K_{64} \cdot \hat{T}_c(t) + K_{74} \cdot \hat{T}_1(t). \end{cases} \tag{17}$$

TABLE I. RELATIONSHIPS BETWEEN COMPENSATORS, PVS, AND VAS

Compensator	PV	AV
$G_{c1}(s)$	$h_1(t)$	Valve A
$G_{c2}(s)$	$T_o(t)$	Valve C
$G_{c3}(s)$	$H_2(t)$	Valve B

IV. CONTROL SYSTEM DESIGN

As mentioned above, with the linear model in (12) representative of the process, it is possible to use any linear control technique to design the necessary compensators for the correct operation of the process. As a first design step, a process control diagram will be used to identify the different compensators to be implemented

Fig. 3 shows a sort of piping and instrumentation diagram (P&ID) [6] of the process in question. In this P&ID the compensators can be identified, as the measured process variables, and also the actuation signals. Three control goals have been identified in the process. Namely, to regulate the levels $h_1(t)$ and $h_2(t)$, and to control the outlet temperature $T_o(t)$. Therefore, these variables represent the process variables (PV) of interest. The actuators (AV) of the process are identified as the three control valves, i.e., valves A, B, and C. Therefore, the compensators to be designed are identified in the P&ID by their respective transfer functions in the Laplace domain, i.e. $G_{c1}(s)$, $G_{c2}(s)$, and $G_{c3}(s)$. In Table I, the relationships between the PVs, AVs, and compensators are presented.

From Fig. 3 it is clear, therefore, that the control system to be designed is a multivariable control [7], but isolated from each other, i.e., each of the compensators are feedback output compensators, with a PI structure. In Fig. 4, a diagram with the proposed control structure is presented. From Fig. 4, it can be clearly seen, the presence of the three compensators to be designed, and their relationships with the PVs and AVs. Indeed, the compensator $G_{c1}(s)$ allows regulating $h_1(t)$ by varying $f_A(t)$ (through the controlled throttling of valve A), the level $h_2(t)$ is also controlled through the modification of the flow $f_B(t)$ (controlled throttling of valve B), and finally, the control of $T_o(t)$ through the regulation of $f_C(t)$ (controlled throttling of valve C). At this point, it is evident that the flow $f_C(t)$ acts as the fluid that allows the temperature exchange to the mixture $f_1(t)+f_B(t)$.

Using MATLAB-Sisotool, it was possible to design the compensators with the aid of (20) and Fig. 3 and 4.

V. SIMULATION RESULTS

The simulation model is that shown in (10), which is a lossless and nonlinear system. The simulation was carried out using MATLAB-Simulink. In Fig. 5, the simulation model is presented. The simulation model is comprised of three dynamic blocks and one scope. The process block represents the nonlinear model in (10). The

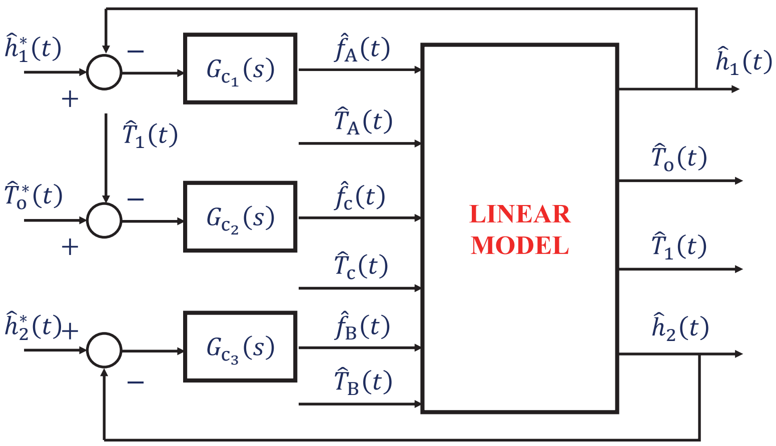

Fig. 4. Proposed closed-loop control structure.

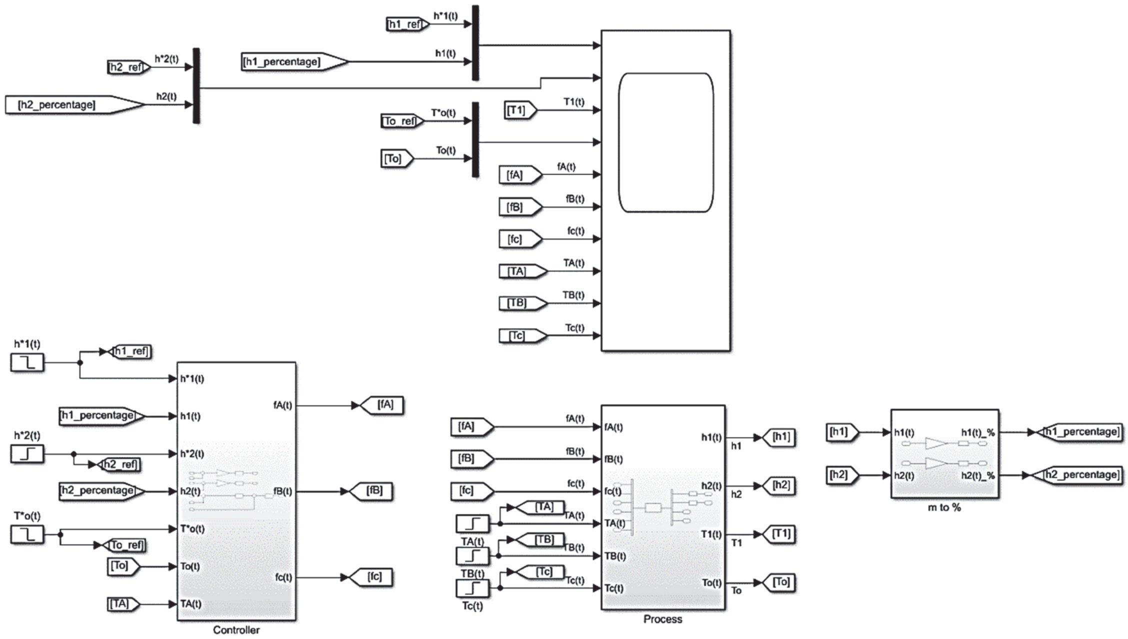

Fig. 5. Simulation model.

controller block groups the three controllers described in Fig. 4. Finally, there is a third block, which represents a change of units, related to the PVs. It should be emphasized that, in most of the practical cases, where level controls are involved, the PVs are measured in % instead of m.

The variation of the set points is of the step type, i.e., $h_1^*(t)$, $h_2^*(t)$, and $T_o^*(t)$ and variations that act as perturbations to the process have also been included. These variations are also of the step type and correspond to the temperatures $T_A(t)$, $T_B(t)$, and $T_C(t)$.

The simulation results are presented in Fig. 6. The initial conditions are assumed to be $h_1^*(t)$ = 90 %, $h_2^*(t)$ = 80 %, and $T_o^*(t)$ = 400 °C. Then at 8.3 min the process requires to decrease the level $h_1(t)$ to 85 %. Subsequently at 13.3 min the level $h_2(t)$ is required to decrease to 85 %. Finally, at 18.3 min, $T_o(t)$ is required to decrease by 20 °C, reaching a new value of 380 °C. Regarding perturbations, the process undergoes a sudden rise of $T_A(t)$ at 25 min from the start of the simulation of 20 °C. Before the perturbation $T_A(t)$ = 60 °C, at 25 min $T_A(t)$ = 80 °C. Then at 31 min, $T_B(t)$ shows an increase of 600 °C. Before this perturbation, $T_B(t)$ = 400 °C and after it, $T_B(t)$ = 1000 °C. The last perturbation is related to $T_C(t)$ and takes place at 42 min. Before the perturbation $T_C(t)$ = 10 °C but then it reaches its new value of $T_C(t)$ = 25 °C.

From Fig. 6, it can be seen that the compensators (which are of the PI type) operate appropriately, rapidly responding both to set point changes and in the presence of perturbations. But they also show high overshoots in response to these changes, which can be significantly reduced by applying a second tuning session. In addition, a negligible steady state error is observed.

VI. Conclusion

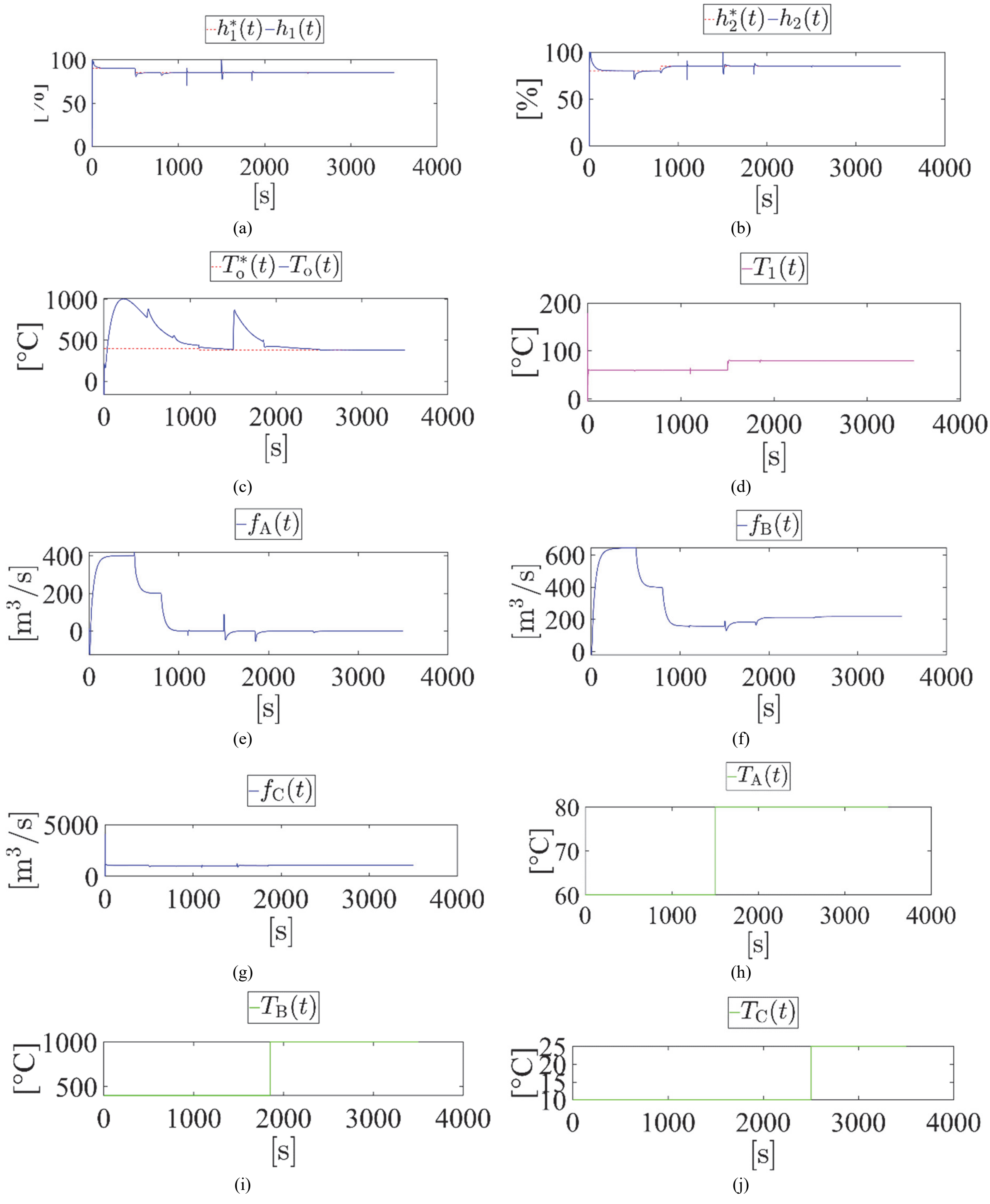

Fig. 6. Simulation results. (a)–(c) show the dynamics of $h_1(t)$, $h_2(t)$, and $T_o(t)$ respectively. (d) shows the dynamic of $T_1(t)$. (e)–(g) show the dynamic of $f_A(t)$, $f_B(t)$, and $f_C(t)$ respectively. (h)–(j) show the dynamic of $T_A(t)$, $T_B(t)$, and $T_C(t)$ respectively.

The knowledge of the dynamics of a process allows a control engineer to obtain the so-called insight of the process, providing him with the necessary confidence to be more agile when it comes to solving failures.

It is in this approach that the present article has described a thermal process, which is implemented by two thermal tanks, connected in series that allows processing different fluids that present different volumetric flows with different thermal capacities.

A systematic procedure for the modeling of a thermal process has been presented, which includes obtaining the nonlinear dynamic equations and the design of the associated compensators. For this purpose, the principles of the laws of conservation of mass and energy are applied to the control volumes existing in the process. Once obtained the

set of nonlinear dynamic equations of the process, we proceed to linearize them, around the operating points where the process operates in steady state, thus obtaining a linear one. Once the linear model of the process is available and the control objectives have been identified, linear control techniques can be applied to design, in a simple way, the compensators involved in the process. In particular, the MATLAB-Sisotool tool has been used for the design of the three compensators involved in the process.

Finally, the model has been simulated using MATLAB-Simulink, which includes the nonlinear equations of the process, as well as the dynamics of the designed compensators.

From the simulation results, it is possible to observe that the designed compensators operate appropriately to the set point changes and to the different perturbations that appear during the simulation. The dynamics of the process variables show fast responses, with negligible steady state errors. However, they show high overshoots, which can be decreased after subjecting the compensators to further tuning sessions.

As an opportunity for improvement of this study, it can be the inclusion of the control parameter of the valve stems (assuming that these valves are with stems) to the dynamic equations of the process and thus, allow the compensators to operate directly on the dynamics of the valves, a situation that occurs in this study, since here, the compensators act directly on the process flows.

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